

Flavors in an expanding plasma

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We consider the effect of an expanding plasma on probe matter by determining time-dependent D7 embeddings in the holographic dual of an expanding viscous plasma. We calculate the chiral condensate and meson spectra including contributions of viscosity. The chiral condensate essentially confirms the expectation from the static black hole. For the meson spectra we propose a scheme that is in agreement with the adiabatic approximation. New contributions arise for the vector mesons at the order of the viscosity terms.

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I. INTRODUCTION

The study of the properties of strongly interacting quark-gluon plasma (QGP) [1] is one of the most investigated research topics in recent years. The interest is fueled on the one hand by direct experimental questions related to the properties of QGP produced at RHIC and on the other hand by the possibility of studying analytically the non-perturbative properties of plasmas in various, mostly supersymmetric, gauge theories using the AdS/CFT correspondence [2]. Although there does not exist so far a direct counterpart of the QCD plasma, the study of exact properties of similar systems not based on *ad-hoc* phenomenological models may increase our understanding of the properties of the QCD plasma. This hope has been substantiated, e.g., by the discovery of a universal strong coupling shear viscosity to entropy ratio, which is valid for a wide range of different gauge theories [3, 4].

A lot of work has been initially undertaken for a static plasma system with a fixed constant temperature. After the earliest investigations of transport coefficients various other observables have been discussed related to the properties of fundamental flavors in the thermal medium such as drag force calculations [5, 6], jet-quenching [7, 8, 9], meson spectrum [10, 11], meson melting [12], thermodynamics of fundamental flavors [13].

On the other hand, since the experimentally produced plasma is always a non-static expanding system, it was tempting to extend the AdS/CFT investigations to such a time-dependent dynamical setting. Qualitative duals for thermalization and cooling have been suggested in [14, 15], a quantitative framework for studying boost-invariant expansion of a plasma system in $\mathcal{N} = 4$ SYM has been proposed in [16]. Subsequent work within this framework include [17, 18, 19, 21, 22, 23, 24, 25, 26], which concentrated on the details of the dynamics of the expanding plasma in *pure* $\mathcal{N} = 4$ SYM theory.

The main motivation for this work was to bring together the two lines of investigation and to study an expanding plasma in the $\mathcal{N} = 4$ SYM theory with fundamental flavors. The flavor system is represented by an embedding of a D7 brane in the dual geometry of the expanding plasma, which is – in an essential way – time-dependent. Such time-dependent embeddings have not been considered so far in the literature on flavor systems. In this paper we would like to concentrate on the features of the system that are the direct analogues of the corresponding properties studied in the static case. In particular we find the (time-dependent) embedding, we find

the time-dependence of the chiral condensate including leading viscosity effects and we describe the behavior of mesonic modes in the scalar and vector channels. Finally we perform a holographic renormalization of the D7 action density. We will show that the chiral condensate, meson spectra and action density are compatible with the adiabatic approximation, i.e. their leading contribution agrees with the result obtained from the static AdS/Blackhole by naively assuming Bjorken scaling.

As a word of caution let us emphasize that the gauge theory that we are considering does not exhibit spontaneous chiral symmetry breaking so from this point of view differs substantially from QCD. However, since this is the first investigation of a flavor system in an expanding plasma, we chose to deal with the simplest theoretical setting, which is the only system for which the dual expanding geometry is known so far. Ultimately one would like to extend these investigations to theories that exhibit chiral symmetry breaking.

The plan of this paper is as follows. We will first review some basic facts about viscous hydrodynamics from field theory perspective and its implementation in AdS/CFT. In the next section we will perturbatively determine the time-dependent embedding of a D7 brane in that geometry and determine the chiral quark condensate. Thereafter, we determine meson spectra from fluctuations about the embedding. Moreover, we give the regularized D7 action and show that it is again compatible with expectations from adiabatic considerations.

A. Boost invariant kinematics

An interesting kinematical regime of the expanding plasma is the so-called central rapidity region. There, as was suggested by Bjorken [27], one assumes that the system is invariant under longitudinal boosts. This assumption is in fact commonly used in realistic hydrodynamic simulations of QGP [28]. If in addition we assume no dependence on transverse coordinates (a limit of infinitely large nuclei) the dynamics simplifies enormously.

In order to study boost-invariant plasma configurations it is convenient to pass from Minkowski coordinates (x^0, x^1, x_\perp) to proper-time/spacetime rapidity ones (τ, y, x_\perp) through

$$x^0 = \tau \cosh y \quad x^1 = \tau \sinh y \quad (1)$$

The object of this work is to describe the spacetime dependence of the energy-momentum tensor of a boost-invariant

plasma in $\mathcal{N} = 4$ SYM theory at strong coupling. The symmetries of the problem reduce the number of independent components of $T_{\mu\nu}$ to three. Energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ and tracelessness $T^\mu_\mu = 0$ allows to express all components in terms of just a single function – the energy density $\varepsilon(\tau)$ in the local rest frame. Explicitly we have [16]

$$T_{\tau\tau} = \varepsilon(\tau) \quad (2)$$

$$T_{yy} = -\tau^2 \left(\varepsilon(\tau) + \frac{d}{d\tau} \varepsilon(\tau) \right) \quad (3)$$

$$T_{xx} = \varepsilon(\tau) + \frac{1}{2} \tau \frac{d}{d\tau} \varepsilon(\tau) \quad (4)$$

Gauge theory dynamics should now pick out a definite function $\varepsilon(\tau)$.

B. Viscous hydrodynamics

The object of a hydrodynamic model is to determine the spacetime dependence of the energy-momentum tensor for an expanding (plasma) system. The simplest dynamical assumption is that of a perfect fluid. This amounts to assuming that the energy momentum has the form

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p\eta_{\mu\nu} \quad (5)$$

where u^μ is the local 4-velocity of the fluid ($u^2 = -1$), ε is the energy density and p is the pressure. In the case of $\mathcal{N} = 4$ SYM theory that we consider here $T^\mu_\mu = 0$ and hence $\varepsilon = 3p$. The equation of motion that one obtains from energy conservation in the boost-invariant setup is

$$\partial_\tau \varepsilon = -\frac{\varepsilon + p}{\tau} \equiv -\frac{4}{3} \frac{\varepsilon}{\tau} \quad (6)$$

whose solution is the celebrated Bjorken result

$$\varepsilon = \frac{\varepsilon_0}{\tau^{\frac{4}{3}}} \quad (7)$$

Once one wants to include dissipative effects coming from shear viscosity, the description becomes more complex. In a first approximation one adds to the perfect fluid tensor a dissipative contribution $\eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu)$, where η is the shear viscosity of the fluid. The resulting equations of motion get modified to

$$\partial_\tau \varepsilon = -\frac{4}{3} \frac{\varepsilon}{\tau} + \frac{4\eta}{3\tau^2} \quad (8)$$

Note that in the above equation the shear viscosity is generically temperature dependent ($\eta \propto T^3$ in the $\mathcal{N} = 4$ case) and hence τ dependent. In order to have a closed system of equations we have to incorporate this dependence through

$$\eta = \eta_0 \cdot \varepsilon^{\frac{3}{4}} \quad (9)$$

with η_0 being some numerical coefficient. The resulting energy density satisfying (8) gets modified from (7) to

$$\varepsilon(\tau) = \frac{\varepsilon_0}{\tau^{\frac{4}{3}}} \cdot \left(1 - \frac{2\eta_0}{\varepsilon_0^{\frac{1}{4}} \tau^{\frac{2}{3}}} + \dots \right). \quad (10)$$

C. AdS/CFT description of an expanding boost-invariant plasma

In [16, 17, 18, 19] the program of perturbatively determining the dual geometry to the viscous hydrodynamic model discussed in the previous section was carried out. The non-compact part of the metric takes the form

$$\begin{aligned} \frac{ds^2}{L^2} &= \frac{1}{z^2} \tilde{g}_{\mu\nu} dX^\mu dX^\nu + \frac{dz^2}{z^2} \\ \tilde{g}_{\mu\nu} dX^\mu dX^\nu &= \left(-e^{\mathcal{A}(z,\tau)} d\tau^2 + e^{\mathcal{B}(z,\tau)} \tau^2 dy^2 + e^{\mathcal{C}(z,\tau)} dx_\perp^2 \right) \end{aligned} \quad (11)$$

with z the holographic direction. The energy-momentum tensor is related to the fourth order term of $\tilde{g}_{\mu\nu}$ expanded in z [32].

$$\begin{aligned} \langle T_{\mu\nu} \rangle &= \frac{N_c^2}{2\pi^2} \lim_{z \rightarrow 0} \frac{1}{z^4} (\tilde{g}_{\mu\nu} - \eta_{\mu\nu}) \\ \Rightarrow \varepsilon(\tau) &= -\frac{N_c^2}{2\pi} \lim_{z \rightarrow 0} \frac{\mathcal{A}(z,\tau)}{z^4} \end{aligned} \quad (12)$$

Of course it is too difficult to perform this construction for arbitrary functions $\varepsilon(\tau)$. What has been performed in practice is an expansion of $\varepsilon(\tau)$ for (large) proper-times τ [16, 17, 18, 19].

Each subsequent term in the expansion can then be determined by requiring regularity of the square of the Riemann tensor order-by-order. In [16] it was shown that in order to study the large proper-time limit of the metric one is led to introduce a scaling variable

$$v = \frac{z}{\tau^{\frac{1}{3}}} \varepsilon_0^{\frac{1}{4}} \quad (13)$$

and take the limit $\tau \rightarrow \infty$ with v fixed. For the discussion of subleading terms in the metric an expansion around this limit was performed,

$$\mathcal{A}(z,\tau) = a_0(v) + a_1(v) \frac{1}{\varepsilon_0^{\frac{1}{4}} \tau^{\frac{2}{3}}} + \dots, \quad (14)$$

and similarly for the other coefficients. The leading and first subleading coefficients are given by

$$\begin{aligned} a_0(v) &= \ln \frac{(1-v^4/3)^2}{1+v^4/3}, & a_1(v) &= 2\eta_0 \frac{(9+v^4)v^4}{9-v^8}, \\ b_0(v) &= \ln(1+v^4/3), & b_1(v) &= -2\eta_0 \frac{v^4}{3+v^4} + 2\eta_0 \ln \frac{3-v^4}{3+v^4}, \\ c_0(v) &= \ln(1+v^4/3), & c_1(v) &= -2\eta_0 \frac{v^4}{3+v^4} - \eta_0 \ln \frac{3-v^4}{3+v^4}. \end{aligned} \quad (15)$$

η_0 is an undetermined integration constant (which has the physical interpretation as the coefficient of shear viscosity). It can be fixed from non-singularity of the metric; the resulting value

$$\eta_0 = \frac{1}{2^{\frac{1}{2}} 3^{\frac{3}{4}}} \quad (16)$$

is in accord with the known viscosity coefficient of $\mathcal{N} = 4$ SYM, $\eta/s = 1/4\pi$, calculated in the static case [3].

From the leading order terms in (15), using the similarity with the static black hole metric, we may read off the temperature from the position of the horizon. As discussed in [18], to this order, the result

$$T(\tau) = \left(\frac{4\varepsilon_0}{3} \right)^{\frac{1}{4}} \frac{1}{\pi \tau^{\frac{1}{3}}} \left[1 - \frac{\eta_0}{2\varepsilon_0^{1/4} \tau^{2/3}} \right] \quad (17)$$

is compatible with the Stefan-Boltzmann law. This can be explicitly checked by extracting the energy density from the metric expanded in the scaling limit up to $\mathcal{O}(\tau^{-2})$ through (12). We obtain

$$\varepsilon(\tau) = \frac{N_c^2}{2\pi^2} \cdot \frac{\varepsilon_0}{\tau^{\frac{4}{3}}} \cdot \left(1 - \frac{2\eta_0}{\varepsilon_0^{\frac{1}{3}} \tau^{\frac{2}{3}}} + \dots\right), \quad (18)$$

which is consistent with viscous hydrodynamic evolution (10).

II. TIME-DEPENDENT D7 EMBEDDING

A. D7 embeddings

Conventional AdS/CFT describes $\mathcal{N} = 4$ SYM theory, which has only fields in the adjoint representation, since all fields sit in the same supermultiplet containing the gauge field. A standard method [30] for the introduction of quenched matter into the correspondence is by adding probe D7 branes. Strings stretching between the probe and the original D3 stack generating the $AdS_5 \times S^5$ background give rise to an $\mathcal{N} = 2$ hypermultiplet in the fundamental representation.

Let us consider geometries $M_5 \times S^5$ with M_5 asymptotically AdS_5 and line element

$$\begin{aligned} ds_{10}^2 &= \frac{r^2}{L^2} ds_4^2 + \frac{L^2}{r^2} dr^2 + d\Omega_5^2 \\ &= \frac{r^2}{L^2} ds_4^2 + \frac{L^2}{r^2} d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2 \end{aligned} \quad (19)$$

with $r^2 = \sum (X^i)^2 = \rho^2 + (X^8)^2 + (X^9)^2 = 1/z^2$.

Placing a D7 probe parallel to the first eight coordinates, its position and shape are described by the coordinates X^8 and X^9 . Manifestly, the D7 brane breaks the $SU(4) \simeq SO(6)$ symmetry of the internal manifold to $SO(4) \times SO_{89}(2) \simeq SU(2)_L \times SU(2)_R \times U(1)_R$. The $SO(2)$ symmetry in the 8,9-plane can be used to rotate the embedding of the D7 to

$$X^8 = 0, \quad X^9 = \Phi(\rho, \tau), \quad (20)$$

such that one scalar field is sufficient for a complete description of the embedding.

The embedding $\Phi(\rho, \tau)$ is determined from the D7 action,

$$\begin{aligned} S_{D7} &= \mu_7 \int d^8 \xi e^{-\varphi} \sqrt{\det P[g]_{ab} + F_{ab}} \\ &\quad + \mu_7 \int d^8 \xi P[(\partial_\rho C_4)] \varepsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma, \end{aligned} \quad (21)$$

where $P[\dots]$ denotes the pull-back to the world-volume.

While $AdS_5 \times S^5$ permits constant embeddings $\Phi \equiv m$, more general geometries require a non-trivial profile $\Phi(\rho)$. To our knowledge the present article is the first-time that also time-dependence for the embedding has been considered in the context of flavored AdS/CFT.

Close to the boundary the embedding behaves as

$$\Phi \xrightarrow{\rho \rightarrow \infty} m + \frac{c}{\rho^2} + \dots, \quad (22)$$

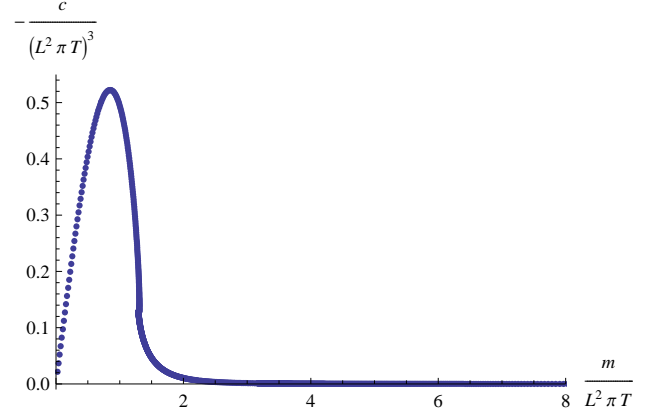


FIG. 1: Chiral condensate as function of the quark mass for the static AdS/Schwarzschild background.

where m and c are related to the bare quark mass m_q and chiral condensate $\langle \mathcal{O} \rangle$ by [13]¹

$$m_q = \frac{m}{2\pi\alpha'}, \quad (23)$$

$$\langle \mathcal{O} \rangle = -\frac{N_f N_c}{(2\pi\ell_s^2)^3 \lambda} c, \quad (24)$$

where $\lambda = g_{YM}^2 N_c = 2\pi g_s N_c = L^4/(2\ell_s^4)$. For the AdS/Schwarzschild geometry such a VEV forms, but vanishes for $m_q \rightarrow 0$ such that there is no *spontaneous* chiral symmetry breaking associated to its appearance [11].

The common procedure for finding the embedding is to derive the equations of motion from (21) and impose the requirement that the embedding have an interpretation as a holographic renormalization group flow. In particular this means that $\Phi(\rho)$ should be one-valued as a function of the holographic energy scale r , which requires regularity as $\rho \rightarrow 0$. (This condition is not sufficient however as will be discussed below.) Because of the parity symmetry $r \mapsto -r$ of the underlying geometry, which manifests itself as $\rho \mapsto -\rho$ invariance of the equation of motion, embedding solutions are either parity odd, known as ‘black hole’ solution since they hit the horizon, or even, i.e., $\partial_\rho \Phi = 0$, known as ‘Minkowski-type’ solutions. It is easy to show that the condition for Minkowski-type solutions corresponds to the absence of a conical defect on the brane [29].

B. AdS/Schwarzschild: Adiabatic Approximation

Figure 1 shows the chiral condensate as a function of the quark mass for the AdS/Schwarzschild geometry. Since all quantities enter the equations of motion in units of the black hole mass m_{bh} (we express everything in terms of $\pi T = 8G_N/(3\pi)m_{bh}$), the leading order time-dependence of the chiral condensate can be determined in an adiabatic approximation using the fact that the perfect fluid geometry for

¹ To be more precise these are the mass of the $\mathcal{N} = 2$ hypermultiplet and the vacuum expectation value of the operator $\mathcal{O} = \bar{\psi}\psi + q^\dagger \Phi q + m_q q^\dagger q$.

constant τ appears like an AdS/Schwarzschild solution with fixed temperature (17).

Our numerical results provide $c/T^3 \sim (m/T)^\alpha$. α is about -6 at $m \approx 3(\pi T)$ and increases to -5.2 when going to $m \approx 8(\pi T)$. In other words

$$c \sim \tau^{-(3-\alpha)/3} \approx \tau^{-8.2/3} \quad (25)$$

However, as has been observed in [29], the numerics determining Figure 1 is not particularly accurate for large quark mass because the chiral condensate becomes small. Therefore it is not possible to increase the quark mass till the exponent saturates.

For large quark mass, the solutions are very far from the black hole and become approximately constant embeddings as in the supersymmetric scenario [30]. This suggests the following perturbative analysis [13]. For small ϵ we seek regular solutions

$$\Phi(\rho) = m + \epsilon f(\rho) \quad (26)$$

of the DBI action in AdS/Schwarzschild geometry [11].²

$$\mathcal{L}_{D7/AdS/BH} = \rho^3 \left(1 - \frac{\epsilon(L^2 \pi T)^8}{16(\rho^2 + \Phi(\rho)^2)^4} \right) \sqrt{1 + \Phi'(\rho)} \quad (27)$$

We obtain

$$f(\rho) = -\frac{(L^2 \pi T)^8 (3m^4 + 3\rho^2 m^2 + \rho^4)}{96m^5 (m^2 + \rho^2)^3}, \quad (28)$$

$$\Rightarrow \frac{c}{(L^2 \pi T)^3} = -\frac{1}{96} \left(\frac{L^2 \pi T}{m} \right)^5. \quad (29)$$

We have written (29) in a way that emphasizes the known fact that the temperature can be effectively removed from the equations by a suitable redefinition of the quark mass and condensate. In the present context, such a rescaling is not desirable. With (17) we conclude that the adiabatic estimate for the perfect fluid geometry is

$$c = -\frac{\epsilon_0^2 L^{16}}{54m^5} \tau^{-\frac{8}{3}}. \quad (30)$$

We will check in the following that this is indeed the leading contribution for the expanding plasma geometry.

² For conformity with our conventions, we have replaced their w_6 by Φ and the parameter b by $L^2 \pi T$, with T the temperature of the black hole.

C. Viscous fluid

Restricted to the scalar Φ , the action reads for our geometry (11)

$$S_{D7} = \mathbf{N} \int d\tau d\rho \tau \rho^3 \mathbf{A} \sqrt{1 + \Phi'^2 - \mathbf{B} \frac{\dot{\Phi}^2}{(\rho^2 + \Phi^2)^2}}, \quad (31)$$

$$\mathbf{A} := \left(1 - \frac{v^8}{9}\right) \exp \left[-2\eta_0 \epsilon_0^{-\frac{1}{4}} \frac{v^8}{9 - v^8} \tau^{-\frac{2}{3}} \right], \quad (32)$$

$$\mathbf{B} := \frac{1 + \frac{v^4}{3}}{(1 - \frac{v^4}{3})^2} \exp \left[2\eta_0 \epsilon_0^{-\frac{1}{4}} v^4 \frac{9 + v^4}{9 - v^8} \tau^{-\frac{2}{3}} \right],$$

$$v := \frac{\epsilon_0^{\frac{1}{4}} L^2}{\tau^{\frac{1}{3}} \sqrt{\rho^2 + \Phi(\rho, \tau)^2}}, \quad (33)$$

$$\mathbf{N} := N_f T_{D7} \Omega_3 V_x = \frac{1}{2} \frac{N_c N_f}{(2\pi \ell_s^2)^4 \lambda} V_x, \quad (34)$$

where $T_{D7} = 2\pi/(g_s(2\pi\ell_s)^8)$ is the D7 tension. $\Omega_3 = 2\pi^2$ and $V_x = \int dy d^2 x_\perp$ are the volume of the unit three-sphere and spatial part of the boundary, respectively. The latter is of course infinite.

Since the viscous fluid geometry discussed in the previous section behaves similar to AdS/Schwarzschild with time-dependent temperature, we do not expect spontaneous symmetry breaking either, though going to small quark masses leaves the domain of validity of the geometry and it is thus hard to make a definite statement. We will therefore only consider ‘Minkowski-type’ embeddings that avoid the horizon at the center of the geometry.

For a given quark mass, in general regularity is only possible for a discrete set of values for the chiral condensate. In the regime under consideration, since no phase transition occurs, we expect $c = c(m)$ to be a one-valued function.

The equation of motion arising from (31) is a non-linear partial differential equation. We will solve it perturbatively by a late-time expansion

$$\Phi(\rho, \tau) = m + \sum_{i=1}^{\infty} f_i(\rho) \tau^{-\frac{i}{3}}. \quad (35)$$

We use a fraction of $1/3$ in the exponent because all exponents showing up in the background geometry (11) are integer multiples of one third. The ansatz reduces the equations of motion to the following (infinite) system of ordinary differential equations

$$\rho^{-3} \partial_\rho (\rho^3 f'_i(\rho)) = \mathcal{I}_i(\rho) \quad (36)$$

$$\mathcal{I}_i = \frac{8m\epsilon_0^2}{9(m^2 + \rho^2)^5} \cdot \begin{cases} 1 & \text{if } i = 8 \\ -4\eta_0 \epsilon_0^{-1/4} & \text{if } i = 11 \\ 0 & \text{else; provided } i < 14 \end{cases}$$

The boundary behavior of solutions to (36) is

$$f_i(\rho) \xrightarrow{\rho \rightarrow \infty} m_i + \frac{c_i}{\rho^2}, \quad (37)$$

which becomes an exact solution when the inhomogeneous term vanishes, $\mathcal{I}_i = 0$. The first term, m_i , contributes $m_i \tau^{-i/3}$ to the bare quark mass. Since we do not accept a time-dependence of the bare parameters on physical grounds, we require $m_i = 0$. Thus in conjunction with regularity the

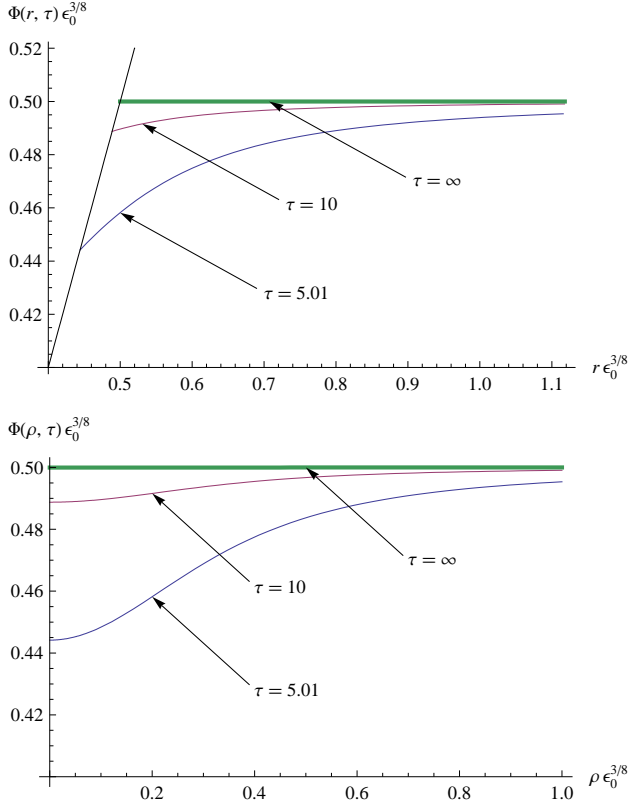


FIG. 2: Embeddings with $m = \frac{1}{2}\epsilon_0^{-3/8}$ at different times. For late times (bold), the supersymmetric embedding is approximated. The diagonal line in the first plot corresponds to $\Phi(r) \equiv r$, where the embeddings have to end.

value of c_i is completely fixed. In particular $\mathcal{I}_i = 0$ implies $c_i = 0$ or $f_i \equiv 0$.

To the considered order the solution is

$$\Phi(\rho, \tau) = m + c \frac{\rho^4 + 3\rho^2 m^2 + 3m^4}{(m^2 + \rho^2)^3}, \quad (38)$$

with

$$c = -\frac{\epsilon_0^2 L^{16}}{54m^5} \tau^{-8/3} \left(1 - 4\eta_0 \epsilon_0^{-1/4} \tau^{-2/3} + \dots \right). \quad (39)$$

In figure 3 we show the effect of the subleading term on the shape of $c(\tau)$. Note that we can trust this result only as long as this term is small compared to the leading order,

$$4\eta_0 \epsilon_0^{-1/4} \tau^{-2/3} \ll 1. \quad (40)$$

We observe that the viscosity term enters the chiral condensate exactly the same way as it enters the energy density squared. We may therefore express the chiral condensate as³

$$\langle \mathcal{O} \rangle = \frac{1}{216\pi^4} \frac{N_f \lambda^3}{N_c^3} \frac{\epsilon^2}{m_q^5}. \quad (41)$$

³ We would like remind the reader that $\epsilon(\tau) \sim T^4 N_c^2$, such that $\langle \mathcal{O} \rangle \sim N_c N_f T^8$.

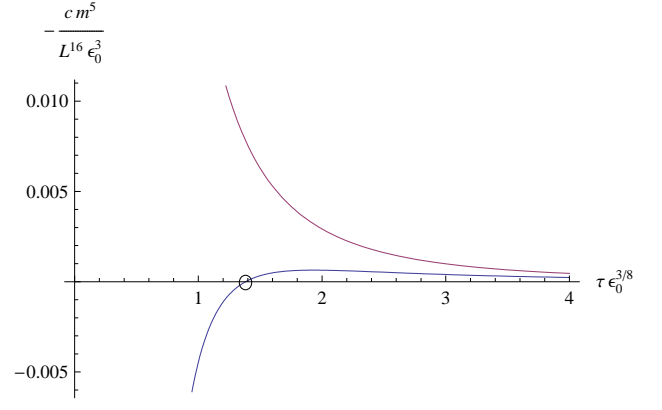


FIG. 3: Time-dependence of the chiral condensate for adiabatic approximation (monotonic curve) and with viscosity correction. The circle at $\tau = (4/3)^{9/8} \approx 1.38$ is outside the regime of validity given by equation (40).

This solution is however only valid for late times as each term in $\epsilon(\tau)$ apparently diverges for $\tau \rightarrow 0$. While at this stage it is not clear up to which time (if at all) our perturbative expansion converges, we may still ask for which range of τ it is self-consistent. For this consideration, three time-scales are of potential importance. Firstly, the time when the solution touches the horizon, i.e., $\Phi(0, \tau) = 3^{-1/4} L^2 \epsilon_0^{1/3} \tau^{-1/3}$. Secondly, the moment when the viscosity term in the expansion dominates the leading term in such a way that the embedding “recoils” and stops being a one-valued function of the holographic direction r . This happens when $d^2 \Phi(r, \tau)/dr^2 \rightarrow \infty$. Thirdly, the time $\tau = (4/3)^{9/8} \approx 1.38$ when the chiral condensate changes its sign and by (41) would lead to an imaginary energy density. Before this time scale is reached, the regime of validity (40) is left.

For numerical computations and plots, we will use $\epsilon_0^{-3/8}$ (which is a length) as a unit to express dimensionful quantities. Figure 2 shows the $m = 1/2\epsilon_0^{-3/8}$ solution at various times before break-down. (Again we would like to stress that the solutions may be invalid even before – by “break-down” we denote their having become invalid for sure.) Figure 4 compares the three effects. For small quark mass solutions are invalidated by touching the horizon, and by imaginary energy density for large quark mass.

III. MESON SPECTRA

In the D3/D7 framework, meson spectra are determined from regular, normalizable solutions to the equations obtained from linearizing the full equations of motion of the D7 brane about the embedding solution that describes the position and shape of the brane [10].

In the following section we distinguish between four dimensional meson modes, which carry a “4d” label and eight dimensional fluctuations, which always start with a δ followed by a (Greek or Latin) capital letter, e.g. $\delta\Phi$ or δA^y . Our ansätze are products of spherical harmonics \mathcal{Y} on the internal manifold, wave forms parallel to the boundary and radial parts, which describe the dependence on the holographic coordinates and are denoted by δ followed by a small letter.

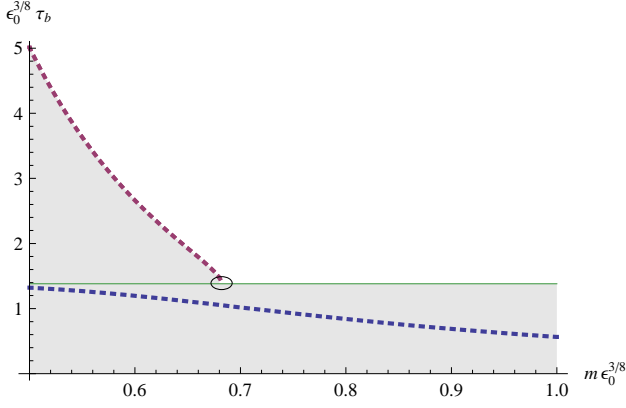


FIG. 4: Break-down time of the solution as a function of the quark mass. The horizontal line indicates the time where the energy density becomes imaginary, the dashed flat curve is where embeddings become two-valued as a function of the holographic energy scale r and the steep curve shows where solutions touch the horizon, i.e., can no longer be considered ‘Minkowski-type’. The gray area marks invalid solutions. The circle is at $m \approx 0.682 \epsilon_0^{-3/8}$, which is the smallest mass that does not lead to solutions that eventually touch the horizon.

A. Boost invariance for the AdS geometry

Before turning to the actual holographic computation of meson spectra for the viscous fluid, it is insightful to investigate how boost invariance changes wave forms of scalar and vector mesons in the conventional setting, where there is no time-dependence of the meson mass.

In four dimensional Minkowski space the massive Klein-Gordon equation assumes the form

$$\square \Phi_{4d} = \left[-\frac{1}{\tau} \partial_\tau \tau \partial_\tau + \tau^{-2} \partial_y^2 + \partial_x^2 \right] \Phi_{4d} = M^2 \Phi_{4d} \quad (42)$$

$$\Rightarrow \Phi_{4d} = (c_1 J_0(\omega \tau) + c_2 Y_0(\omega \tau)) e^{\pm i k_\perp x_\perp} \quad (43)$$

$$k_\perp^2 = k_2^2 + k_3^2$$

with J_0 and Y_0 Bessel functions of first kind. Since a linear combination of the Bessel functions will appear frequently in our expressions we introduce the short hand

$$\mathcal{F}_p[\omega] := c_1 J_p(\int \omega d\tau) + c_2 Y_p(\int \omega d\tau). \quad (44)$$

We will not explicitly denote the time-dependence of \mathcal{F}_0 arising from the integral over τ . At this stage, the integral has been chosen for later convenience and gives $\omega \tau$ for constant frequencies. The eigenfrequencies $\omega := \sqrt{M^2 + k_\perp^2}$ are to be determined in our holographic setup. We will thus assume $k_\perp = 0$ from the start to obtain the mass spectrum.

The 4d meson field is given as the boundary value of (linear, normalizable) fluctuations $\delta\Phi$, $\delta\Psi$ about the embedding solution $\Phi(\rho) \equiv m$,

$$X^8 = 0 + \delta\Psi(\rho, \tau), \quad X^9 = \Phi(\rho) + \delta\Phi(\rho, \tau). \quad (45)$$

For the presentation of our ansatz, we will concentrate on the scalar mode $\delta\Phi$; the pseudoscalar mode $\delta\Psi$ will be treated analogously. With the following holographic ansatz

$$\delta\Phi(\rho, \tau) = \delta\phi(\rho) \mathcal{F}_0[\omega] \mathcal{Y}^\ell(S^3), \quad (46)$$

the boundary value corresponding to quantum number ℓ is defined by

$$\Phi_{4d}^{(\ell)} = \lim_{\rho \rightarrow \infty} \rho^2 \frac{\delta\Phi(\rho, \tau)}{\mathcal{Y}^\ell(S^3)} \quad (47)$$

Equation (46) is a natural modification of the ansatz given in [10] to separate the D7 equation of motion in anti-de Sitter space:

$$\left[-\frac{L^2}{(\rho^2 + m^2)^2} \frac{1}{\tau} \partial_\tau \tau \partial_\tau + \frac{1}{\rho^3} \partial_\rho \rho^3 \partial_\rho + \frac{1}{\rho^2} \Delta_{S^3} \right] \delta\Phi(\rho, \tau) = 0. \quad (48)$$

The radial equation obtained after separation reads

$$\left[\frac{1}{\rho^3} \partial_\rho \rho^3 \partial_\rho + \frac{L^2 \omega^2}{\rho^2 + m^2} - \frac{\ell(\ell+2)}{\rho^2} \right] \delta\phi(\rho) = 0, \quad (49)$$

which is the well-known result of [10]. For simplicity, we will only consider the lowest Kaluza-Klein mode on the internal S^3 , such that $\ell = 0$, $\mathcal{Y}^0 \equiv 1$.

The requirements of regularity in the interior ($\rho \rightarrow 0$) and vanishing at the boundary ($\rho \rightarrow \infty$), fix the modes completely. One obtains a discrete set of modes the lightest of which is given by

$$\mathcal{F}_0 \left[\omega_0 = \frac{\sqrt{8m}}{L^2} \right] \cdot \frac{1}{m^2 + \rho^2} \quad (50)$$

For a four-dimensional massive vector meson we have

$$\nabla_a F^{ab} = M^2 A_{4d}^b. \quad (51)$$

We assume that the solutions are still plane waves in the x_2, x_3 -plane, i.e., $A_{4d}^a = \xi^a(\tau) \exp i k_\perp x_\perp$. This yields the following component equations

$$-\tau \partial_\tau \left(\frac{1}{\tau} \partial_\tau A_y^{4d} \right) = (M^2 + k_\perp^2) A_y^{4d} \quad (52)$$

$$-\partial_\tau^2 A_2^{4d} - \frac{1}{\tau} \partial_\tau A_2^{4d} + i k_2 \left(\partial_\tau + \frac{1}{\tau} \right) A_\tau^{4d} \quad (53)$$

$$+ k_2 k_3 A_3^{4d} = (M^2 + k_3^2) A_2^{4d} \quad \text{and } (2 \leftrightarrow 3)$$

$$A_\tau^{4d} = -\frac{i \partial_\tau (k_2 A_2^{4d} + k_3 A_3^{4d})}{\omega^2} \quad (54)$$

Equation (52) can be treated separately. Its solution is

$$A_y^{4d} = \tau \mathcal{F}_1[\omega] e^{i k_\perp x_\perp} \quad A_{\tau, x_2, x_3}^{4d} = 0. \quad (55)$$

The others may be solved without loss of generality by turning the coordinate system such that $k_3 = 0$. Then it follows immediately that $A_{2,3}^{4d} = \xi_{2,3} \mathcal{F}_0[\omega_{2,3}] \exp i k_\perp x_\perp$. With this modified ansatz and plugging (54) into (53) we obtain

$$\left[-M^2 - k_3^2 + \omega_2^2 \left(1 - \frac{k_2^2}{\omega^2} \right) \right] A_2^{4d} + k_2 k_3 \left(1 - \frac{\omega_3^2}{\omega^2} \right) A_3^{4d} = 0$$

and $(2 \leftrightarrow 3)$, (56)

which can only be satisfied for $\omega_2 = \omega_3 =: \omega_{23}$. Moreover, since it is a homogeneous system, $\omega_{23}(M, k_2, k_3)$ can be determined from degeneracy of the coefficient matrix. We shall not reproduce the final expression, but just note that $\omega_{23} = M$ when $k_2 = k_3 = 0$, which could also have been obtained directly from (53). We thus end up with the two solutions

$$A_{2,3}^{4d} = \xi_{2,3} \mathcal{F}_0[\omega_{23}] \exp i k_\perp x_\perp. \quad (57)$$

Therefore, we adapt the holographic ansätze for meson modes found in [10] as follows

Type

$$\begin{aligned}
\text{I} \quad & \delta A_\alpha = \delta a_I^\pm(\rho) \mathcal{F}_0[\omega] e^{ik_\perp x_\perp} \mathcal{Y}_\alpha^{\ell, \pm}(S^3), \quad \alpha = 5, 6, 7; \\
\text{II}_y \quad & \delta A_y = \delta a_{IIy}(\rho) \tau \mathcal{F}_1[\omega] e^{ik_\perp x_\perp} \mathcal{Y}^\ell(S^3); \\
\text{II}_{2,3} \quad & \delta A_2 = \delta a_{II2}(\rho) \mathcal{F}_0[\omega] e^{ik_\perp x_\perp} \mathcal{Y}^\ell(S^3), \quad A_3 = 0; \\
& \text{and } (2 \leftrightarrow 3) \\
\text{III} \quad & \delta A_\rho = \delta a_{III}(\rho) \mathcal{F}_0[\omega] e^{ik_\perp x_\perp} \mathcal{Y}^\ell(S^3), \\
& \delta A_\alpha = \delta \tilde{a}_{III}(\rho) \mathcal{F}_0[\omega] e^{ik_\perp x_\perp} \mathcal{Y}_\alpha^{\ell, \pm}(S^3); \quad (58)
\end{aligned}$$

with the respective other components set to zero. We will only consider modes of type II, which are the only modes dual to vector mesons and therefore most interesting.

In the ansätze, the dependence in the 0, 1, 2, 3 directions has been modified as compared to what can be found in [10]. The reason these changes are straight-forward is the following: The calculation of [10] only uses two important properties of the ansätze regarding derivatives in those directions,

$$\Delta_{4d} \delta A_I = M^2 \delta A_I, \quad I \in [0, \dots, 7] \quad (59)$$

$$g_{4d}^{ab} \partial_a \delta A_b = 0, \quad (60)$$

where $g_{4d} = \text{diag}(-1, \tau^2, 1, 1)$ in our case, whereas in [10] it was a Minkowski metric. For our ansätze, the gauge condition (60) is either trivially obeyed or follows from (54). Moreover it can be used to turn (59) into (51).

B. Viscous fluid geometry

Before coming to the actual holographic computation, we would like to discuss the general framework of late-time perturbative expansions that we use.

Since the viscous fluid geometry and our D7 embeddings are time-dependent, we do not expect, and do not see, a separation into a purely τ dependent and ρ dependent factor. This makes the problem very difficult to tackle analytically. We are helped by the property that at late proper times the geometry becomes pure AdS_5 with the corresponding D7 brane embedding. In this limit the simplest solution looks like (50)

$$\mathcal{F}_0 \left[\omega_0 = \frac{\sqrt{8m}}{L^2} \right] \cdot \frac{1}{m^2 + \rho^2}, \quad (61)$$

where $\mathcal{F}_0[\omega]$ is defined in equation (44). For smaller proper-times it is natural to treat the frequency appearing in (61) as depending on τ . However as the equations do not allow for a separation of variables we have τ dependence also in the remaining part:

$$\tilde{\mathcal{F}}[\omega(\tau)] f(\rho, \tau) \quad (62)$$

where we allow for a general $\tilde{\mathcal{F}}$ which should reduce to $\mathcal{F}_0[\omega]$ for *constant* ω . We have moreover the expansions

$$\omega(\tau) = \omega + \frac{1}{\tau^{\frac{1}{3}}} \omega^{(1)} + \dots \quad (63)$$

$$f(\rho, \tau) = f^{(0)}(\rho) + \frac{1}{\tau^{\frac{1}{3}}} f^{(1)}(\rho) + \dots \quad (64)$$

Note that the above form is not unique. Redefining the coefficients of the expansions in an appropriate way, we may

redefine the split (62). So in order to uniquely specify such an ansatz we have to supplement the usual regularity condition at $\rho = 0$ and Dirichlet boundary condition at $\rho = \infty$ by another condition which makes the split (62) unique. In this paper we will impose a condition on the profile of the mode $\delta\phi$ induced on the boundary

$$\Phi_{4d}(\tau) \equiv \lim_{\rho \rightarrow \infty} \rho^2 \delta\phi(\rho, \tau) \quad (65)$$

Namely we will set

$$\Phi_{4d}(\tau) = \sqrt{\frac{\int \omega_{4d}(\tau) d\tau}{\omega_{4d} \tau}} \mathcal{F}_0[\omega_{4d}(\tau)]. \quad (66)$$

which provides a definition of our frequency $\omega_{4d}(\tau)$. For constant $\omega_{4d}(\tau)$ this reduces of course to the pure AdS_5 result (61).

Our motivation for the above form (66) is that it arises as a WKB approximation to a Klein-Gordon equation with time dependent mass spectra:

$$\square \Phi_{4d} = \left[-\frac{1}{\tau} \partial_\tau \tau \partial_\tau + \tau^{-2} \partial_y^2 + \partial_x^2 \right] \Phi_{4d} = M_{4d}^2(\tau) \Phi_{4d} \quad (67)$$

We may separate variables by assuming a plane wave in the 2, 3 plane and obtain $\omega_{4d}^2(\tau) = M_{4d}^2(\tau) + k_\perp^2$. (Though we will assume $k_\perp = 0$, henceforth.) The remaining equation

$$-\frac{1}{\tau} \partial_\tau \tau \partial_\tau \Phi_{4d}(\tau) = \omega_{4d}^2(\tau) \Phi_{4d}(\tau) \quad (68)$$

can only be solved approximately, e.g., by the WKB approximation, which gives two linearly independent solutions,

$$\Phi_{4d}(\tau) \approx \sqrt{\frac{\int \omega_{4d}(\tau) d\tau}{\omega_{4d} \tau}} \mathcal{F}_0[\omega_{4d}(\tau)]. \quad (69)$$

The square root prefactor ensures that Abel's theorem is fulfilled, such that the Wronskian for our ansatz is

$$W = \frac{\text{const}}{t} \quad (70)$$

as it should be for the exact solution.

We have now all ingredients in place to actually calculate the meson spectrum for the time-dependent viscous fluid ge-

ometry. We expand the D7 action (21),⁴ given by

$$\mathcal{L}_{DBI} = e^{\frac{A}{2} + \frac{B}{2} + C} \rho^3 \tau \left[1 + (\partial_\rho X^9)^2 + (\partial_\rho X^8)^2 + e^{-C} (\partial_\rho A_2)^2 + \frac{e^{-B}}{\tau^2} (\partial_\rho A_y)^2 - \frac{e^{-A} L^4}{r^4} (I) + (\text{quartic}) \right]^{1/2}$$

$$(I) = (\partial_\tau X^8)^2 + (\partial_\tau X^9)^2 + (\partial_\rho X^8)^2 (\partial_\tau X^9)^2 + (\partial_\tau X^8)^2 (\partial_\rho X^9)^2 - 2(\partial_\tau X^8)(\partial_\rho X^9)(\partial_\rho X^8)(\partial_\tau X^9) + e^{-C} (II) + \frac{e^{-B}}{\tau^2} (III) \quad (71)$$

$$(II) = (\partial_\tau A_2)^2 + (\partial_\rho X^9)^2 (\partial_\tau A_2)^2 + (\partial_\tau X^9)^2 (\partial_\rho A_2)^2 - 2(\partial_\tau X^9)(\partial_\rho X^9)(\partial_\rho A_2)(\partial_\tau A_2) \quad (72)$$

$$(III) = +(\partial_\tau A_y)^2 + (\partial_\rho X^9)^2 (\partial_\tau A_y)^2 + (\partial_\tau X^9)^2 (\partial_\rho A_y)^2 - 2(\partial_\tau X^9)(\partial_\rho X^9)(\partial_\rho A_y)(\partial_\tau A_y) \quad (73)$$

to quadratic order in fluctuations

$$X^9 = \Phi + \delta\Phi, \quad X^8 = 0 + \delta\Psi, \\ A_2 = 0 + \delta A_2, \quad A_y = 0 + \delta A_y. \quad (74)$$

The resulting equation of motion is evaluated by performing a perturbative expansion in $\tau^{-1/3}$,

$$\delta\Phi = c_1 J_0 \left(\int \omega^{(\phi)}(\tau) d\tau \right) \sum_{j=0}^{\infty} \delta\phi_j(\rho) \tau^{-j/3} + c_2 Y_0 \left(\int \omega^{(\phi)}(\tau) d\tau \right) \sum_{j=0}^{\infty} \delta\tilde{\phi}_j(\rho) \tau^{-j/3}, \quad (75)$$

$$\omega^{(\phi)} = \sum_{i=0}^{\infty} \omega_i \tau^{-\frac{i}{3}}, \quad (76)$$

and analogously for the other fluctuations.⁵ We use the known asymptotic expansion of the Bessel functions and obtain schematically the following equation

$$[\text{polynomial in } \tau^{-1/3}] \cos \left(\int \omega^{(\phi)} d\tau \right) + [\text{polynomial in } \tau^{-1/3}] \sin \left(\int \omega^{(\phi)} d\tau \right) = 0. \quad (77)$$

At any given order, the requirement that the coefficients of the polynomials vanish, provides a differential equation for $\delta\phi_i$ and $\delta\tilde{\phi}_i$ depending on ω_i (and lower order solutions). We have to go to order 6 before the viscosity η_0 enters the equations. To this order, the equations for $\delta\phi_i$ and $\delta\tilde{\phi}_i$ can be separated by choosing suitable linear combinations and yield $\delta\phi_i \equiv \delta\tilde{\phi}_i$, which is what is required for the WKB ansatz (69) to be applicable. We impose the boundary conditions

$$\delta\phi \xrightarrow{\rho \rightarrow \infty} 0, \quad \delta\phi \xrightarrow{\rho \rightarrow 0} \text{finite}, \\ \rho^2 \delta\phi \xrightarrow{\rho \rightarrow \infty} \sqrt{\frac{\int \omega^{(\phi)} d\tau}{\omega^{(\phi)} \tau}} \mathcal{F}_0[\omega^{(\phi)}]. \quad (78)$$

⁴ We do not write out those quartic terms that can only produce terms quartic in fluctuations.

⁵ The equation of motion of A_y requires a slightly modified ansatz given in the appendix.

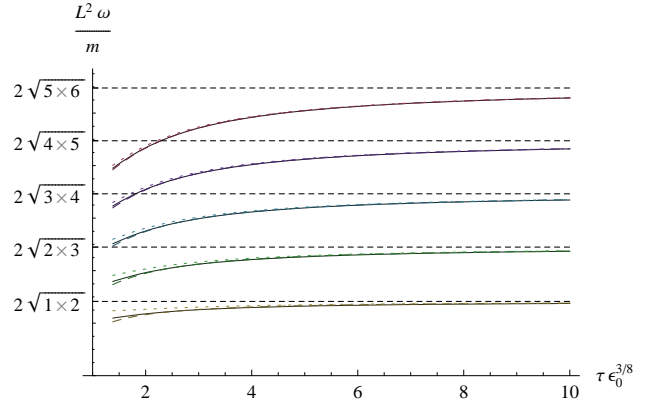


FIG. 5: Late-time spectra for the viscous fluid geometry. The supersymmetric spectrum is shown as dashed horizontal lines. Scalar mesons are shown as continuous lines, vector modes are dashed.

The first two of these conditions pick regular normalizable solutions; the last ensures that meson solutions on the boundary (65) satisfy the constraint (70). Consequently, the conditions fix two integration constants and the frequency ω_i at each order in the perturbative expansion. The only remaining free constants are the overall factors c_1 and c_2 of our ansatz (75).

Each of the coefficient functions has to satisfy a differential equation that is best expressed with the substitutions

$$\delta\phi_i(\rho) = (1 - \mathbf{y})^{-n-1} \delta\phi_i(\mathbf{y}), \\ \mathbf{y} = -\rho^2/m^2, \\ \omega_0^2 = m^2((2n+3)^2 - 1). \quad (79)$$

The lowest order equation then reads

$$[\mathbf{y}(1 - \mathbf{y})\partial_{\mathbf{y}}^2 + (\mathbf{c} - (\mathbf{a} + \mathbf{b} - 1)\mathbf{y})\partial_{\mathbf{y}} - \mathbf{a}\mathbf{b}] \phi_0(\mathbf{y}) = 0 \\ \mathbf{a} = -n - 1, \quad \mathbf{b} = -n, \quad \mathbf{c} = 2. \quad (80)$$

This is exactly the hypergeometric equation already encountered in [10]. The boundary conditions fix n to be a non-negative integer, thus yielding a discrete meson spectrum $M = \omega_0(n)$ and the solutions in terms of (degenerate) hypergeometric functions ${}_2F_1$ are

$$h_1 = {}_2F_1(-n-1, -n, 2; \mathbf{y}), \quad (81)$$

$$h_2 = (1 - \mathbf{y})^{3+2n} {}_2F_1(n+2, n+3, 2n+4; 1 - \mathbf{y}). \quad (82)$$

Only h_1 is regular, such that $\delta\phi_0 \equiv h_1$.

Higher orders in perturbation theory produce inhomogeneous terms in the analogues of (80). Since it is a linear ordinary equation, the solution can still be obtained in closed form by standard methods. However, the resulting integrals are hard to solve in general. Since both solutions $h_{1,2}$ are rational functions of \mathbf{y} and $\ln \mathbf{y}$, it is however easy to do so for definite n . For the lowest five mesons $n = 0, \dots, 4$ we give the solutions in the appendix.

The mass⁶ of the lowest scalar meson mode is

$$\omega^{(\phi)} = \frac{4\pi}{\sqrt{\lambda}} \cdot \left[m_q - \frac{3\lambda^2 \varepsilon_0}{80\pi^4 \tau^{4/3} m_q^3} \cdot \left(1 - \frac{2\eta_0}{\tau^{2/3} \varepsilon_0^{1/4}} \right) \right]. \quad (83)$$

To the considered order, pseudoscalar modes $\delta\psi$ have exactly the same equations of motion and the spectrum is degenerate. Moreover we note that the spectrum agrees with the adiabatic approximation even including the viscosity corrections. The reason for this might be that the bulk metric coefficients that enter the calculation for the scalar mesons can be expressed completely in terms of the energy density, whereas the components for the $y, 2, 3$ directions cannot.

The vector mesons deviate slightly from the scalar modes. For comparison we plot the mass ratio of scalar and vector modes in Figure 7. The mass of the lowest vector mesons is given by

$$\omega^{(A^y)} = \frac{4\pi}{\sqrt{\lambda}} \cdot \left[m_q - \frac{7\lambda^2 \varepsilon_0}{240\pi^4 \tau^{4/3} m_q^3} \cdot \left(1 - \frac{6\eta_0}{7\tau^{2/3} \varepsilon_0^{1/4}} \right) \right], \quad (84)$$

$$\omega^{(A^{2,3})} = \frac{4\pi}{\sqrt{\lambda}} \cdot \left[m_q - \frac{7\lambda^2 \varepsilon_0}{240\pi^4 \tau^{4/3} m_q^3} \cdot \left(1 - \frac{18\eta_0}{7\tau^{2/3} \varepsilon_0^{1/4}} \right) \right]. \quad (85)$$

This agrees with the adiabatic approximation excluding the viscosity term. Since the metric components that enter the holographic computation, g_{yy} and g_{22} , agree only up to the viscosity terms, this deviation does not come as a surprise.

We plot the five lowest meson modes in Figure 5. The leading order term gives the exact supersymmetric spectrum that is approached for $\tau \rightarrow \infty$.

C. Comparison to the adiabatic approximation

In this subsection, we will review some properties of low-temperature meson spectra for the static AdS black hole. Plugging in the time-dependence of the temperature into the static meson spectra, yields an estimate for the time-dependent spectrum, which we will refer to as *adiabatic approximation*. We will assume that the temperature dependence given in terms of Poincaré time t can be obtained from (17) by substituting τ for t :

$$T_{AdS/BH}(t) = \left(\frac{4\varepsilon_0}{3} \right)^{1/4} \frac{1}{\pi t^{1/3}} \left[1 - \frac{\eta_0}{2\varepsilon_0^{1/4} t^{2/3}} \right]. \quad (86)$$

Note however that we do not expect the resulting adiabatic meson spectra to accurately give the viscosity corrections. The reason for this is that even though the horizon position can be expressed completely in terms of the energy density, such that the Stefan–Boltzmann law holds, the bulk metric and energy momentum tensor nevertheless contain additional viscosity terms that cannot be captured by the AdS/Schwarzschild geometry even when the geometry near the horizon and near the boundary is matched.

In Figure 6 we plot the numerical solution (dots) of the static case. We calculate the asymptotic solution in a low temperature expansion (solid curves), which agrees with the

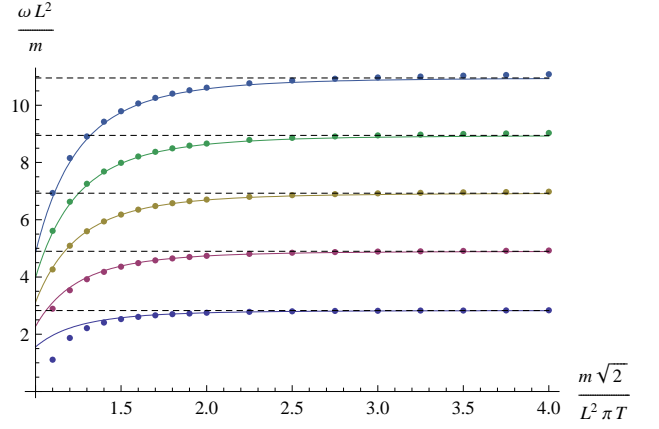


FIG. 6: The plot shows the (pseudo)scalar meson spectrum. For small temperatures, the supersymmetric spectrum (dashed) is approached. The solid lines are asymptotic $T \rightarrow 0$ solutions, which are in good agreement with the numerical solutions (dots) for small temperatures.

numerical calculation. Plugging the time dependence of the temperature (86) into this analytical approximation, we obtain the time-dependent meson spectrum in adiabatic approximation. The mass of the lowest scalar and vector modes are given by

$$\begin{aligned} \omega_{\phi,\psi}^{\text{ad.}} &= \frac{4\pi}{\sqrt{\lambda}} \left[m_q - \frac{9\lambda^2 T^4}{320 m_q^3} \right] \\ &= \frac{4\pi}{\sqrt{\lambda}} \left[m_q - \frac{3\lambda^2}{80\pi^4} \frac{\varepsilon_0}{m_q^3} t^{-4/3} \left(1 - \frac{2\eta_0}{t^{2/3} \varepsilon_0^{1/4}} \right) \right], \end{aligned} \quad (87)$$

$$\begin{aligned} \omega_{A^\mu}^{\text{ad.}} &= \frac{4\pi}{\sqrt{\lambda}} \left[m_q - \frac{7\lambda^2 T^4}{320 m_q^3} \right] \\ &= \frac{4\pi}{\sqrt{\lambda}} \left[m_q - \frac{7\lambda^2}{240\pi^4} \frac{\varepsilon_0}{m_q^3} t^{-4/3} \left(1 - \frac{2\eta_0}{t^{2/3} \varepsilon_0^{1/4}} \right) \right]. \end{aligned} \quad (88)$$

The mass of all of these modes decreases for increasing temperature.⁷ Moreover, the adiabatic scalar modes completely agree with our result (83), whereas the vector modes only agree in the leading contribution; the viscosity terms differ. We consider the agreement of the scalars accidental in the sense that it is a consequence of a certain property of the expanding plasma geometry: All metric coefficients entering the scalar equation of motion can be expressed purely in terms of the temperature, while additional viscosity terms only show up in those metric coefficients that end up in the equations for the vector modes.

An important assumption in our calculation has been that the Klein–Gordon equation is obeyed by scalar particles. This assumption can actually be proved employing the holographic equation of motion resulting from the linearization procedure. There is a parity symmetry $\rho \mapsto -\rho$ in the equation of motion. Since only ‘Minkowski-type’ solutions are considered, this will

⁶ defined by equation (66)

⁷ A similar decrease of meson masses for increasing (static) temperature was found in the Sakai/Sugimoto model [33].

lead to even solutions, which have the following expansion,

$$\delta\Phi(\rho \rightarrow \infty, \tau) \sim \Phi_{4d}(\tau) \frac{1}{\rho^2} - \frac{L^4 \Phi_{4d}(\tau)}{8} M^2(\tau) \frac{1}{\rho^4} + \dots, \quad (89)$$

because only normalizable solutions are allowed, such that the constant leading term vanishes. The subleading coefficient $M^2(\tau)$ has been multiplied by an additional factor $-L^4 \Phi_{4d}/8$ for later convenience. Plugging above expansion into the eight-dimensional equation of motion, yields at leading order in $1/\rho$,

$$-\frac{1}{\tau} \partial_\tau (\tau \partial_\tau) \Phi_{4d} = M^2(\tau) \Phi_{4d}. \quad (90)$$

This establishes that at least for a background geometry dual to a hydrodynamic expansion up to and including viscosity, the scalar meson equation is a Klein–Gordon equation with time-dependent mass. This might change when also encoding higher order effects like the relaxation time [19], which currently appears to be out of reach of a supergravity approximation [20].

We will now assess the error of the WKB approximation by plugging our meson solutions into the four dimensional Klein–Gordon equation. The error should be smaller than τ^{-2} to be subleading to the viscosity contribution. We first determine the four dimensional meson solution by

$$\Phi_{4d}^{(\ell)} = \lim_{\rho \rightarrow \infty} \rho^2 \delta\phi(\rho, \tau) \mathcal{F}_0[\omega^{(\phi)}] \quad (91)$$

where on the right hand side we plug in the mass (83) of the lowest holographic scalar meson solution, i.e., we set $\omega_{4d} \approx \omega^{(\phi)}$.

With (91) the error estimate $\Delta\omega$ can be obtained from the Klein–Gordon equation

$$\frac{1}{\tau} \partial_\tau \tau \partial_\tau \Phi_{4d}(\tau) = (\omega^{(\phi)}(\tau) + \Delta\omega(\tau))^2 \Phi_{4d}(\tau), \quad (92)$$

by linearizing in $\Delta\omega$. This yields

$$\Delta\omega(\tau) = \frac{\sqrt{2} L^{10} \varepsilon_0}{5m^4} \frac{1}{\tau^{10/3}} + \dots, \quad (93)$$

which is sufficiently small, that is subleading to the viscosity terms arising from the geometry. (Also note that the frequencies $\omega^{(\phi)}(\tau)$ and the meson mass obtained from the holographic expansion (89) agree up to and including order τ^{-2} .) However when encoding hydrodynamic effects in the geometry that are of sufficiently high order, we would be forced to consider a better approximation for our ansatz, e.g., by using higher order WKB. Moreover, beyond a certain order the WKB ansatz is expected not to work anymore because the coefficients $\delta\phi_i$ and $\delta\tilde{\phi}$ are not expected to coincide to arbitrary order.

IV. REGULARIZED D7 ACTION

In the static case, the free energy density can be related to the regularized D7 action by means of a Wick rotation. In the conventions of [13] the time direction is periodically identified with $\beta = 1/T$, such that the free energy density F is given by

$$F = T \cdot \frac{S_{D7}}{V_x}, \quad (94)$$

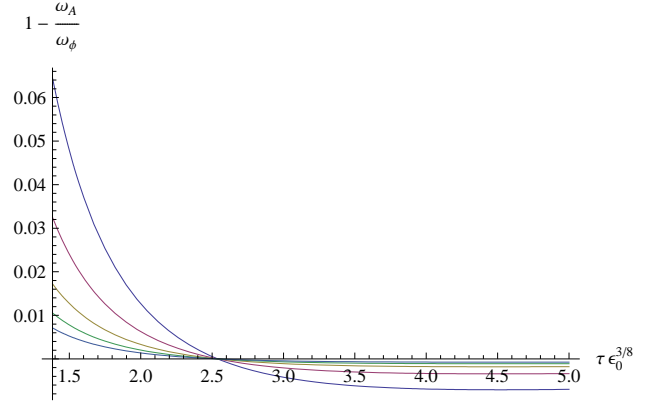


FIG. 7: The plot shows the relative difference of the masses of (type II_y) vector and scalar mesons for $m = \varepsilon_0^{3/8}$.

with T the temperature and $V_x = \int d^3x$ the (infinite) spatial volume of the boundary. The authors of [13] obtain

$$\frac{S_{D7}}{V_x} \sim -\frac{N_c N_f T^3 \lambda}{32} \cdot \frac{1}{12} \left(\frac{T}{2m_q / \sqrt{\lambda}} \right)^4 = -\frac{N_c N_f \lambda^3}{6114 m_q^4} T^7 \quad (95)$$

in the limit of low temperature. While it is clear that for a time-dependent background neither $1/T = \int dt$ nor (94) really make sense, we are still interested in how our action relates to above asymptotic result.

When calculating the action S_{D7} , the integration along the holographic direction ρ on the brane is encumbered by UV divergences. The standard procedure of holographic renormalization for the case of a probe D7 brane has been worked out in [31]. It consists in regularization by introducing a cut off ρ_{max} and addition of suitable counterterms formed from the induced metric and corresponding curvature on the slice $\rho = \rho_{max}$.

$$S_{reg} = \int_0^{\rho_{max}} \mathcal{L}_{D7} d\rho + \sum L_i(\rho_{max}) \quad (96)$$

To be more precise, the procedure is usually formulated in terms of the coordinate

$$z = (\rho^2 + \Phi^2)^{-1/2}, \quad (97)$$

where the counterterms L_i read

$$\begin{aligned} L_1 &= -\frac{1}{4} \sqrt{\gamma} \\ L_2 &= -\frac{1}{48} \sqrt{\gamma} R_\gamma \\ L_3 &= -\ln(z_{min}) \sqrt{\gamma} \frac{1}{32} (R_{ij} R^{ij} - \frac{1}{3} R_\gamma) \\ L_4 &= \frac{1}{2} \sqrt{\gamma} \Psi^2 \\ L_5 &= -\frac{1}{2} \ln(z_{min}) \Psi (\partial_\tau \gamma^{\tau\tau} \sqrt{\gamma} \partial_\tau + \frac{1}{6} \sqrt{\gamma} R_\gamma) \Psi \\ L_f &= \alpha \gamma \Psi^4 \end{aligned} \quad (98)$$

L_f is a finite counter term, with an arbitrary parameter α , that corresponds to different renormalization schemes. It can

be fixed in supersymmetric settings by the requirement that the free energy vanishes. γ is the induced metric on the $z = z_{min}$ slice and

$$\Psi = \arcsin\left(\frac{z\Phi}{L^2}\right) \quad (99)$$

is the embedding coordinate of the D7 expressed as an angle on the internal S^5 .

We express the counterterms in terms of $\rho_{max} := (z_{min}^{-2} - \Phi^2(\rho_{max}))^{-1/2}$ using equations (99) and (97)

$$\sum L_i = -\frac{1}{4}\tau\rho_{max}^4 + \tau m^4 \frac{5+12\alpha}{12} + \mathcal{O}(\rho_{max}^{-1}) \quad (100)$$

As a check we turn this back into z coordinates by self-consistently iterating (97)

$$\rho_{max}^2 = \sqrt{z_{min}^{-2} + \Phi^2\left(\sqrt{z_{min}^{-2} + \Phi^2\left(\sqrt{z_{min}^{-2} + \Phi^2(\dots)}\right)}\right)} \quad (101)$$

and using the D7 embedding (38). We obtain

$$\frac{1}{\tau} \sum L_i = -\frac{1}{4}(m^2 - z_{min}^{-2})^2 - 4mc + \frac{m^4(5+12\alpha)}{12}, \quad (102)$$

which up to the τ -dependence arising from our rapidity coordinates is exactly the result of [13], eq. (4.21), for $\alpha = -5/12$.

The renormalized action is thus

$$S_{ren} = N \int d\tau \lim_{\rho_{max} \rightarrow \infty} \int_0^{\rho_{max}} (\mathcal{L}_{DBI} - \rho^3 \tau) d\rho + \frac{1}{12} m^4 (5+12\alpha) \tau, \quad (103)$$

where the last two terms are the divergent part of the counterterm, suitably rewritten as an integral, and the remaining finite part. Of course due to the renormalization, the limit can actually be carried out. For late times we obtain

$$S_{ren} = N \int d\tau \tau \left[\frac{m^4}{12} (12\alpha + 5) - \frac{\varepsilon_0^2 L^{16}}{108 m^4 \tau^{8/3}} + \frac{\varepsilon_0^{7/4} \eta_0 L^{16}}{27 m^4 \tau^{10/3}} \right] \quad (104)$$

such that for $\alpha = -5/12$ the configuration indeed relaxes to the supersymmetric setting.

The above is our result. For comparison with (95) we perform an additional, somewhat ill-defined step, namely replacing $\int d\tau \tau \mapsto 1/T$. With $L^4 = 2\lambda \ell_s^4$ and equation (34) we obtain for the supersymmetric scheme

$$S_{ren} = -\frac{1}{3 \cdot 2^{11}} \frac{N_c N_f \lambda^3}{m_q^4} T^7, \quad (105)$$

which agrees with (95).

V. CONCLUSIONS

The main goal of this article was to study fundamental fields in the holographic dual of an expanding viscous fluid. The dual geometry has strong similarity to a black hole geometry with time-dependent temperature. Small temperatures correspond to late-times. We determined the D7 embedding and calculated the consequences of dynamical temperature for the chiral condensate to three orders. The leading order gives the supersymmetric solution, the subleading corresponds to the adiabatic approximation and the subsubleading order includes viscosity corrections going beyond the adiabatic approximation.

Moreover we calculated the meson spectrum and find that it agrees in the subleading order, though only the scalar mesons agree in the subsubleading order with the adiabatic approximation. The agreement crucially depends on the choice of ansatz defining the frequencies. We demonstrate that for our WKB ansatz solves the Klein-Gordon up to an error smaller than that inescapably introduced in the late-time expansion of the geometry.

However, for the next order, which introduces the relaxation time [19], higher order terms would be needed in the WKB approximation in order to still keep that error smaller than the geometry's.

It would be interesting to consider analogous properties for gauge theories which exhibit chiral symmetry breaking and hence are more closer to QCD. However up till now there is no description of an expanding plasma system in such a theory.

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APPENDIX A: MESON SOLUTIONS

(Pseudo-)scalar mesons

$$\begin{aligned}
\delta\Phi_0 &= \mathcal{F}_0[\omega_0] \left[\frac{(8m^4 + 9\rho^2 m^2 + 3\rho^4)\varepsilon_0 L^8}{10m^4(m^2 + \rho^2)^3 \tau^{4/3}} - \frac{(13m^4 + 12\rho^2 m^2 + 3\rho^4)\varepsilon_0^{3/4} \eta_0 L^8}{10m^4(m^2 + \rho^2)^3 \tau^2} + \frac{1}{m^2 + \rho^2} \right], \\
\omega_0 &= \frac{2\sqrt{2}m}{L^2} + \left(-\frac{3L^6 \tau^{-4/3} \varepsilon_0}{5\sqrt{2}m^3} \right) \times \left(1 - \frac{2\eta_0}{\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
\delta\Phi_1 &= \mathcal{F}_0[\omega_1] \left[\frac{(-12m^6 + 15\rho^2 m^4 + 20\rho^4 m^2 + 5\rho^6)\varepsilon_0 L^8}{14m^4(m^2 + \rho^2)^4 \tau^{4/3}} + \frac{(19m^6 - 35\rho^2 m^4 - 35\rho^4 m^2 - 5\rho^6)\varepsilon_0^{3/4} \eta_0 L^8}{14m^4(m^2 + \rho^2)^4 \tau^2} + \frac{\rho^2 - m^2}{(m^2 + \rho^2)^2} \right], \\
\omega_1 &= \frac{2\sqrt{6}m}{L^2} + \left(-\frac{5\sqrt{3}L^6 \tau^{-4/3} \varepsilon_0}{7\sqrt{2}m^3} \right) \times \left(1 - \frac{2\eta_0}{\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
\delta\Phi_2 &= \mathcal{F}_0[\omega_2] \left[\frac{(26m^8 - 140\rho^2 m^6 - 22\rho^4 m^4 + 55\rho^6 m^2 + 11\rho^8)\varepsilon_0 L^8}{30m^4(m^2 + \rho^2)^5 \tau^{4/3}} \right. \\
&\quad \left. - \frac{(41m^8 - 269\rho^2 m^6 + 121\rho^6 m^2 + 11\rho^8)\varepsilon_0^{3/4} \eta_0 L^8}{30m^4(m^2 + \rho^2)^5 \tau^2} + \frac{m^4 - 3\rho^2 m^2 + \rho^4}{(m^2 + \rho^2)^3} \right], \\
\omega_2 &= \frac{4\sqrt{3}m}{L^2} + \left(-\frac{11L^6 \tau^{-4/3} \varepsilon_0}{5\sqrt{3}m^3} \right) \times \left(1 - \frac{2\eta_0}{\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
\delta\Phi_3 &= \mathcal{F}_0[\omega_3] \left[\frac{(-134m^{10} + 1506\rho^2 m^8 - 1650\rho^4 m^6 - 1045\rho^6 m^4 + 342\rho^8 m^2 + 57\rho^{10})\varepsilon_0 L^8}{154m^4(m^2 + \rho^2)^6 \tau^{4/3}} \right. \\
&\quad + \frac{(211m^{10} - 2784\rho^2 m^8 + 3585\rho^4 m^6 + 1805\rho^6 m^4 - 912\rho^8 m^2 - 57\rho^{10})\varepsilon_0^{3/4} \eta_0 L^8}{154m^4(m^2 + \rho^2)^6 \tau^2} \\
&\quad \left. + \frac{-m^6 + 6\rho^2 m^4 - 6\rho^4 m^2 + \rho^6}{(m^2 + \rho^2)^4} \right], \\
\omega_3 &= \frac{4\sqrt{5}m}{L^2} + \left(-\frac{57\sqrt{5}L^6 \tau^{-4/3} \varepsilon_0}{77m^3} \right) \times \left(1 - \frac{2\eta_0}{\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
\delta\Phi_4 &= \mathcal{F}_0[\omega_4] \left[\frac{(68m^{12} - 1279\rho^2 m^{10} + 3543\rho^4 m^8 - 630\rho^6 m^6 - 1566\rho^8 m^4 + 203\rho^{10} m^2 + 29\rho^{12})\varepsilon_0 L^8}{78m^4(m^2 + \rho^2)^7 \tau^{4/3}} \right. \\
&\quad - \frac{(107m^{12} - 2326\rho^2 m^{10} + 7057\rho^4 m^8 - 1840\rho^6 m^6 - 3161\rho^8 m^4 + 638\rho^{10} m^2 + 29\rho^{12})\varepsilon_0^{3/4} \eta_0 L^8}{78m^4(m^2 + \rho^2)^7 \tau^2} \\
&\quad \left. + \frac{m^8 - 10\rho^2 m^6 + 20\rho^4 m^4 - 10\rho^6 m^2 + \rho^8}{(m^2 + \rho^2)^5} \right], \\
\omega_4 &= \frac{2\sqrt{30}m}{L^2} + \left(-\frac{29\sqrt{5}L^6 \tau^{-4/3} \varepsilon_0}{13m^3} \right) \times \left(1 - \frac{2\eta_0}{\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right)
\end{aligned}$$

Vector meson (Type II_{2,3})

$$\begin{aligned}
(\delta a_{II2})_0 &= \mathcal{F}_0[\omega_0] \left[\frac{(22m^4 + 21\rho^2 m^2 + 7\rho^4)\varepsilon_0 L^8}{30m^4(m^2 + \rho^2)^3 \tau^{4/3}} - \frac{(13m^4 + 12\rho^2 m^2 + 3\rho^4)\varepsilon_0^{3/4} \eta_0 L^8}{10m^4(m^2 + \rho^2)^3 \tau^2} + \frac{1}{m^2 + \rho^2} \right], \\
\omega_0 &= \frac{2\sqrt{2}m}{L^2} + \left(-\frac{7L^6 \tau^{-4/3} \varepsilon_0}{15\sqrt{2}m^3} \right) \times \left(1 - \frac{18\eta_0}{7\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{II2})_1 &= \mathcal{F}_0[\omega_1] \left[\frac{(-118m^6 + 137\rho^2 m^4 + 164\rho^4 m^2 + 41\rho^6)\varepsilon_0 L^8}{126m^4(m^2 + \rho^2)^4 \tau^{4/3}} \right. \\
&\quad \left. + \frac{(19m^6 - 35\rho^2 m^4 - 35\rho^4 m^2 - 5\rho^6)\varepsilon_0^{3/4} \eta_0 L^8}{14m^4(m^2 + \rho^2)^4 \tau^2} + \frac{\rho^2 - m^2}{(m^2 + \rho^2)^2} \right], \\
\omega_1 &= \frac{2\sqrt{6}m}{L^2} + \left(-\frac{41L^6 \tau^{-4/3} \varepsilon_0}{21\sqrt{6}m^3} \right) \times \left(1 - \frac{90\eta_0}{41\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right),
\end{aligned}$$

$$\begin{aligned}
(\delta a_{II2})_2 &= \mathcal{F}_0[\omega_2] \left[\frac{(178m^8 - 880\rho^2 m^6 - 106\rho^4 m^4 + 315\rho^6 m^2 + 63\rho^8)\varepsilon_0 L^8}{180m^4(m^2 + \rho^2)^5 \tau^{4/3}} \right. \\
&\quad \left. - \frac{(41m^8 - 269\rho^2 m^6 + 121\rho^4 m^2 + 11\rho^8)\varepsilon_0^{3/4} \eta_0 L^8}{30m^4(m^2 + \rho^2)^5 \tau^2} + \frac{m^4 - 3\rho^2 m^2 + \rho^4}{(m^2 + \rho^2)^3} \right], \\
\omega_2 &= \frac{4\sqrt{3}m}{L^2} + \left(-\frac{7\sqrt{3}L^6 \tau^{-4/3} \varepsilon_0}{10m^3} \right) \times \left(1 - \frac{44\eta_0}{21\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{II2})_3 &= \mathcal{F}_0[\omega_3] \left[\frac{(-4666m^{10} + 48234\rho^2 m^8 - 52030\rho^4 m^6 - 29975\rho^6 m^4 + 9978\rho^8 m^2 + 1663\rho^{10})\varepsilon_0 L^8}{4620m^4(m^2 + \rho^2)^6 \tau^{4/3}} \right. \\
&\quad + \frac{(211m^{10} - 2784\rho^2 m^8 + 3585\rho^4 m^6 + 1805\rho^6 m^4 - 912\rho^8 m^2 - 57\rho^{10})\varepsilon_0^{3/4} \eta_0 L^8}{154m^4(m^2 + \rho^2)^6 \tau^2} \\
&\quad \left. + \frac{-m^6 + 6\rho^2 m^4 - 6\rho^4 m^2 + \rho^6}{(m^2 + \rho^2)^4} \right], \\
\omega_3 &= \frac{4\sqrt{5}m}{L^2} + \left(-\frac{1663L^6 \tau^{-4/3} \varepsilon_0}{462\sqrt{5}m^3} \right) \times \left(1 - \frac{3420\eta_0}{1663\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{II2})_4 &= \mathcal{F}_0[\omega_4] \left[\frac{(1194m^{12} - 20697\rho^2 m^{10} + 55709\rho^4 m^8 - 10890\rho^6 m^6 - 22928\rho^8 m^4 + 2989\rho^{10} m^2 + 427\rho^{12})\varepsilon_0 L^8}{1170m^4(m^2 + \rho^2)^7 \tau^{4/3}} \right. \\
&\quad - \frac{(107m^{12} - 2326\rho^2 m^{10} + 7057\rho^4 m^8 - 1840\rho^6 m^6 - 3161\rho^8 m^4 + 638\rho^{10} m^2 + 29\rho^{12})\varepsilon_0^{3/4} \eta_0 L^8}{78m^4(m^2 + \rho^2)^7 \tau^2} \\
&\quad \left. + \frac{m^8 - 10\rho^2 m^6 + 20\rho^4 m^4 - 10\rho^6 m^2 + \rho^8}{(m^2 + \rho^2)^5} \right], \\
\omega_4 &= \frac{2\sqrt{30}m}{L^2} + \left(-\frac{427L^6 \tau^{-4/3} \varepsilon_0}{39\sqrt{30}m^3} \right) \times \left(1 - \frac{870\eta_0}{427\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right)
\end{aligned}$$

Vector meson (Type II_y)

Vector mesons of type II_y obey a different equation of motion (52), such that the WKB approximation yields a different result

$$A_y^{4d}(\tau) \approx \sqrt{\frac{\int \omega_{4d}(\tau) d\tau}{\omega_{4d} \tau}} \tau \mathcal{F}_1. \quad (\text{A1})$$

The corresponding holographic ansatz

$$\delta A_y = \delta a_{IIy} \tau J_1(\int \omega d\tau) + \delta \tilde{a}_{IIy} \tau Y_1(\int \omega d\tau) \quad (\text{A2})$$

gives rise to the following solutions for the five lowest mesons:

$$\begin{aligned}
(\delta a_{IIy})_0 &= \tau \mathcal{F}_1[\omega_0] \left[\frac{(22m^4 + 21\rho^2 m^2 + 7\rho^4)\varepsilon_0 L^8}{30m^4(m^2 + \rho^2)^3 \tau^{4/3}} - \frac{(11m^4 + 4\rho^2 m^2 + \rho^4)\varepsilon_0^{3/4} \eta_0 L^8}{10m^4(m^2 + \rho^2)^3 \tau^2} + \frac{1}{m^2 + \rho^2} \right], \\
\omega_0 &= \frac{2\sqrt{2}m}{L^2} + \left(-\frac{7L^6 \tau^{-4/3} \varepsilon_0}{15\sqrt{2}m^3} \right) \times \left(1 - \frac{6\eta_0}{7\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{IIy})_1 &= \tau \mathcal{F}_1[\omega_1] \left[\frac{(-118m^6 + 137\rho^2 m^4 + 164\rho^4 m^2 + 41\rho^6)\varepsilon_0 L^8}{126m^4(m^2 + \rho^2)^4 \tau^{4/3}} \right. \\
&\quad \left. + \frac{(81m^6 - 105\rho^2 m^4 - 77\rho^4 m^2 - 11\rho^6)\varepsilon_0^{3/4} \eta_0 L^8}{42m^4(m^2 + \rho^2)^4 \tau^2} + \frac{\rho^2 - m^2}{(m^2 + \rho^2)^2} \right], \\
\omega_1 &= \frac{2\sqrt{6}m}{L^2} + \left(-\frac{41L^6 \tau^{-4/3} \varepsilon_0}{21\sqrt{6}m^3} \right) \times \left(1 - \frac{66\eta_0}{41\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{IIy})_2 &= \tau \mathcal{F}_1[\omega_2] \left[\frac{(178m^8 - 880\rho^2 m^6 - 106\rho^4 m^4 + 315\rho^6 m^2 + 63\rho^8)\varepsilon_0 L^8}{180m^4(m^2 + \rho^2)^5 \tau^{4/3}} \right. \\
&\quad - \frac{(129m^8 - 621\rho^2 m^6 + 40\rho^4 m^4 + 209\rho^6 m^2 + 19\rho^8)\varepsilon_0^{3/4} \eta_0 L^8}{60m^4(m^2 + \rho^2)^5 \tau^2} + \frac{m^4 - 3\rho^2 m^2 + \rho^4}{(m^2 + \rho^2)^3} \left. \right], \\
\omega_2 &= \frac{4\sqrt{3}m}{L^2} + \left(-\frac{7\sqrt{3}L^6 \tau^{-4/3} \varepsilon_0}{10m^3} \right) \times \left(1 - \frac{38\eta_0}{21\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right),
\end{aligned}$$

$$\begin{aligned}
(\delta a_{IIy})_3 &= \tau \mathcal{F}_1[\omega_3] \left[\frac{(-4666m^{10} + 48234\rho^2 m^8 - 52030\rho^4 m^6 - 29975\rho^6 m^4 + 9978\rho^8 m^2 + 1663\rho^{10})\varepsilon_0 L^8}{4620m^4(m^2 + \rho^2)^6 \tau^{4/3}} \right. \\
&\quad + \frac{(3449m^{10} - 34136\rho^2 m^8 + 40675\rho^4 m^6 + 15535\rho^6 m^4 - 8368\rho^8 m^2 - 523\rho^{10})\varepsilon_0^{3/4} \eta_0 L^8}{1540m^4(m^2 + \rho^2)^6 \tau^2} \\
&\quad \left. + \frac{-m^6 + 6\rho^2 m^4 - 6\rho^4 m^2 + \rho^6}{(m^2 + \rho^2)^4} \right], \\
\omega_3 &= \frac{4\sqrt{5}m}{L^2} + \left(-\frac{1663L^6 \tau^{-4/3} \varepsilon_0}{462\sqrt{5}m^3} \right) \times \left(1 - \frac{3138\eta_0}{1663\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right), \\
(\delta a_{IIy})_4 &= \tau \mathcal{F}_1[\omega_4] \left[\frac{(1194m^{12} - 20697\rho^2 m^{10} + 55709\rho^4 m^8 - 10890\rho^6 m^6 - 22928\rho^8 m^4 + 2989\rho^{10} m^2 + 427\rho^{12})\varepsilon_0 L^8}{1170m^4(m^2 + \rho^2)^7 \tau^{4/3}} \right. \\
&\quad - \frac{(891m^{12} - 14718\rho^2 m^{10} + 40421\rho^4 m^8 - 11920\rho^6 m^6 - 14673\rho^8 m^4 + 3014\rho^{10} m^2 + 137\rho^{12})\varepsilon_0^{3/4} \eta_0 L^8}{390m^4(m^2 + \rho^2)^7 \tau^2} \\
&\quad \left. + \frac{m^8 - 10\rho^2 m^6 + 20\rho^4 m^4 - 10\rho^6 m^2 + \rho^8}{(m^2 + \rho^2)^5} \right], \\
\omega_4 &= \frac{2\sqrt{30}m}{L^2} + \left(-\frac{427L^6 \tau^{-4/3} \varepsilon_0}{39\sqrt{30}m^3} \right) \times \left(1 - \frac{822\eta_0}{427\tau^{2/3} \sqrt[4]{\varepsilon_0}} \right)
\end{aligned}$$

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